



## Mean Waiting Time Approximations in the $G/G/1$ Queue

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*In Memory of Our Colleague Vladimir Kalashnikov*

**Abstract.** It is known that correlations in an arrival stream offered to a single-server queue profoundly affect mean waiting times as compared to a corresponding renewal stream offered to the same server. Nonetheless, this paper uses appropriately constructed  $GI/G/1$  models to create viable approximations for queues with correlated arrivals. The constructed renewal arrival process, called *PMRS (Peakedness Matched Renewal Stream)*, preserves the peakedness of the original stream and its arrival rate; furthermore, the squared coefficient of variation of the constructed PMRS equals the index of dispersion of the original stream. Accordingly, the  $GI/G/1$  approximation is termed *PMRQ (Peakedness Matched Renewal Queue)*. To test the efficacy of the PMRQ approximation, we employed a simple variant of the  $TES^+$  process as the autocorrelated arrival stream, and simulated the corresponding  $TES^+/G/1$  queue for several service distributions and traffic intensities. Extensive experimentation showed that the proposed PMRQ approximations produced mean waiting times that compared favorably with simulation results of the original systems. Markov-modulated Poisson process (MMPP) is also discussed as a special case.

**Keywords:**  $G/G/1$  queue,  $GI/G/1$  queue,  $TES^+/G/1$  queue, mean waiting time, PMRS approximation, MMPP

### 1. Introduction

The bulk of traditional queueing theory has addressed queues whose inter-arrival times and service times are independent renewal processes. Whereas the assumption that cus-